ROULETTE

> **HOUSE ADVANTAGE:**
> 5.3% (DOUBLE-ZERO)
> 2.7% (SINGLE-ZERO)
> **TYPICAL HOLD:**
> 22% (NEVADA)

Roulette is a simple game to play: players merely try to guess which number will occur as the outcome of the spin of a ball around a numbered wheel. There are two types of wheels, the "double-zero," common in the United States, and the "single-zero," favored in Europe.

The double-zero wheel contains 38 pockets, numbered 1 through 36, 0 and 00. Of those numbered 1-36, 18 are red and 18 are black. The 0 and 00 are green. The single-zero wheel lacks the 00, and so has only 37 numbers. The game is played essentially the same regardless of which wheel is used, although the house advantage for the single-zero wheel is about half that of the double-zero game (2.7% vs. 5.3%). Except where otherwise specified, the discussion and analysis that follow are based on the double-zero game.

To play the game, players wager on a number, or combination of numbers. The following table shows the types of bets available, with their payoffs and true odds.

Roulette (Double-Zero Wheel)

Type of Bet	*True Odds*	*Payoff*
Straight-up (1 number)	37 to 1	35 to 1
Split (2 numbers)	36 to 2	17 to 1
Street (3 numbers)	35 to 3	11 to 1
Corner (4 numbers)	34 to 4	8 to 1
Five-Number Bet (5 numbers)	33 to 5	6 to 1
Double Street or Line (6 numbers)	32 to 6	5 to 1
Dozens/Columns (12 numbers)	26 to 12	2 to 1
Red/Black/Odd/Even/High/Low (18 numbers)	20 to 18	1 to 1

The dealer spins the wheel counterclockwise and releases a small ball clockwise on a track on the upper portion of the wheel. When the ball loses its momentum, it drops, bounces, and settles into one of the numbered pockets. Players win or lose depending on whether the winning number was among those they selected with their wager. As can be seen from the table above, the lower the probability of winning a particular wager, the higher the payoff. The casino makes money because the payoff odds are less than the true odds.

MATHEMATICS

The mathematics behind roulette is relatively straightforward, making it one of the easiest games in the casino to analyze. Assuming a balanced wheel, spins constitute independent trials. Every bet in the double-zero game will bring the house 5.26% in the long run, with the one exception of the five-number bet. This anomaly holds a 7.89% house advantage. Thus, the casino can expect to win about 5.3% of all money wagered in double-zero roulette.[54]

To see how the mathematics behind the game of roulette works, consider a $1 straight-up bet on a single number. The probability of winning this wager is 1/38 (since there are 38 numbers on a double-zero wheel), and the probability of losing is 37/38. Notice that on the average this wager will win once and lose 37 times for every 38 spins (hence the 37 to 1 odds in the preceding table). To break even, the player would have to be paid $37 for the one win to compensate for the 37 times he lost. That is, the true odds are 37:1. The payoff, however, is only 35:1, resulting in a loss to the player of $2, or 5.26% of the $38 wagered over the 38 spins. This 5.26% house advantage can be computed using the formula for expected value:[55]

$$EV = (+\$35)(1/38) + (-\$1)(37/38) = -\$0.0526.$$

This expected loss of $0.0526 on a $1 wager represents a house advantage of 5.26%. This, of course, is the same percentage edge that would be obtained regardless of the size of the wager. A $5 single number bet, for example, has a player expectation of –$0.263, or 5.26% of $5.

Expectations and advantage for other types of roulette wagers can be computed in the same way. Except for the five-number bet, all will result in a 5.26% house edge. The calculations for a $25 corner bet and a $50 five-number bet are shown below.

$25 Corner Bet

(Payoff = 8:1 = $200, Probability = 4/38).

$$EV = (+\$200)(4/38) + (-\$25)(34/38) = -\$1.316.$$

$$\textit{House advantage} = \$1.316/\$25 = 0.0526, \text{ or } 5.26\%$$

$50 Five-Number Bet

(Payoff = 6:1 = $300, Probability = 5/38)

$$\textit{Expected Value} = (+\$300)(5/38) + (-\$50)(33/38) = -\$3.947.$$

$$\textit{House advantage} = \$3.947/\$50 = 0.0789, \text{ or } 7.89\%.$$

54 The overall house advantage in single-zero roulette is only 2.7%. The bets and payoffs are essentially the same as in the double-zero game (there is no five-number bet, and there may be a few bets offered that are not in the double-zero game), but there are only 37 numbers on the wheel. In addition, sometimes the "en prison" rule is offered in the single-zero game, lowering the house advantage on even-money bets (odd/even, black/red, high/low) to 1.35%. With this rule, rather than losing an even-money wager when the 0 comes up, it remains for one more spin – in prison. If the desired even-money event occurs on the next spin, the original wager is returned with no gain or loss; otherwise it loses. Occasionally a roughly analogous rule called "surrender" can be found in double-zero games. Under this rule, a player loses only half the wager for even-money bets when 0 or 00 hits. Surrender lowers the house advantage for even-money bets in double-zero roulette to 2.63%.

55 As noted in Chapter 2, the house advantage can also be computed directly from the true odds and payoffs: *House Advantage* = (37–35)/(37+1) = 2/38 = 0.0526, or 5.26%.

No pattern or combination of bets will alter the casino's 5.3% advantage.[56] As an illustration, suppose a player makes a combination bet totaling $40 by putting $20 on red, $10 on column 2, and $5 each on 0 and 00. The possible outcomes are:

- Lose $40, with probability 10/38 (if a black number in columns 1 or 3 occurs – lose all bets),
- Lose $10, with probability 8/38 (if a black number in column 2 occurs – win $20 for column 2 bet, lose $30 for the red, 0, and 00 bets),
- Lose $0, with probability 14/38 (if a red number in columns 1 or 3 occurs – win $20 for the red bet, lose $20 for the 0, 00, and column 2 bets),
- Win $30, with probability 4/38 (if a red number in column 2 occurs – win $20 for the red bet, win $20 for the column 2 bet, lose $10 for the 0 and 00 bets),
- Win $140, with probability 2/38 (if either 0 or 00 occurs – win $175 for the winning single number bet, lose $35 for the red, column 2 and losing single number bets).

The expected value is:

$$\begin{aligned} EV &= (-\$40)(10/38) + (-\$10)(8/38) + (\$0)(14/38) \\ &\quad + (+\$30)(4/38) + (+\$140)(2/38) \\ &= (-\$10.526) + (-\$2.105) + (-\$0.000) \\ &\quad + (+\$3.158) + (+\$7.368) \\ &= -\$2.105. \end{aligned}$$

Dividing by the total amount wagered, $40, yields a 5.26% overall house advantage.

INDEPENDENT TRIALS AND A GAMBLER'S FALLACY

A common "gambler's fallacy" is to believe that black is more likely after a series of consecutive reds (black is "due") while another is that since red is "hot," red is more likely on the next spin. Both, of course, are wrong. The spins of a roulette wheel are random, independent events, and so the outcome of one spin does not affect the outcome of any other spin. If a long string of consecutive red outcomes has occurred, the next spin is no more or less likely to be red. The probability of red is 18/38 on each and every spin.

A flaw in the gambler's fallacy logic is mistaking the short run for the long run. While it is true that the probability of, say, ten consecutive reds is small, over the course of millions and millions of spins there will be many such sequences. If one were to examine all sequences of ten consecutive reds over the course of millions of spins, it would turn out that for about 47.37% (18/38) of these sequences, the next spin in the sequence (the eleventh) would be a black number, and for about 47.37%

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56 If the five-number bet is involved in a combination of bets, the overall house advantage for this combination will increase proportional to the fraction of the wager placed on the five-number bet.

the next spin would be red (the remaining 5.26% would be green). Viewed in the context of the long run, a sequence of consecutive outcomes like this should not be surprising and does not change the probabilities of the various outcomes on subsequent spins.

As a by-product of the independence of outcomes in roulette, the multiplication rule for combining probabilities makes it relatively easy to calculate the probability of a series of consecutive outcomes. For example, the probability of getting two red numbers in two spins is (18/38) x (18/38) = .224. The probability of three consecutive reds is (18/38) x (18/38) x (18/38) = .106. The probability of ten consecutive reds is $(18/38)^{10}$ = .000568, or about 1 in 1,758. Similarly, the probability of getting the number 0 on three consecutive spins is $(1/38)^3$ = .0000182, or 1 in 54,872. The probability of getting three consecutive 8's is also .0000182. However, the probability of getting the same number (any number) on three consecutive spins is 38 x .0000182 = 000693, or 1 in 1,444.

ROULETTE VOLATILITY

As noted in Chapter 1, the actual percentage won in a series of wagers will deviate from the expected win percentage due to statistical fluctuations. The degree of volatility depends on the number of wagers in the series and the type of wager.[57] In roulette, the greatest variation can be expected for the straight-up bet on a single number and the least variation for even-money wagers. This can be seen from the values of the per unit standard deviation for each wager. For the straight-up bet,

$$SD_{per\ unit} = \sqrt{(1/38)(-35-.0526)^2 + (37/38)(+1-.0526)^2} = 5.7626\,.$$

For an even-money wager,

$$SD_{per\ unit} = \sqrt{(18/38)(-1-.0526)^2 + (20/38)(+1-.0526)^2} = 0.9986\,.$$

Other roulette wagers have standard deviation values between these two extremes. All else equal, the larger the standard deviation, the greater the volatility and fluctuations that can be expected in a series of wagers.

> PERCENTAGES NEVER LIE. WE BUILT ALL THESE HOTELS ON PERCENTAGES. YOU CAN LOSE FAITH IN EVERYTHING, RELIGION AND GOD, WOMEN AND LOVE, GOOD AND EVIL, WAR AND PEACE, YOU NAME IT. BUT THE PERCENTAGE WILL ALWAYS STAND FAST.
>
> MARIO PUZO
> FOOLS DIE (1978)

57 The wager amount does not affect the volatility associated with win percentage, although it is a factor when analyzing the volatility of the total win amount. See Chapter 2.