CRAPS

Craps is played by betting on the outcome of a roll of a pair of dice. There are a myriad of possible wagers that can be made, each with different odds, payoff and house advantage. To understand these wagers, familiarity with the probabilities associated with the various outcomes of tossing a pair of dice is helpful.

HOUSE ADVANTAGE:
1.41% (PASS/COME, NO ODDS)
0.61% (PASS/COME, 2X ODDS)
1.52% - 16.67% (OTHER BETS)
TYPICAL HOLD:
13% (NEVADA)

The 36 possible outcomes and the probabilities of each possible total are given in the following table.

Outcomes When Rolling A Pair Of Dice					Total	Probability	
(1,1)						2	1/36
(1,2)	(2,1)					3	2/36
(1,3)	(2,2)	(3,1)				4	3/36
(1,4)	(2,3)	(3,2)	(4,1)			5	4/36
(1,5)	(2,4)	(3,3)	(4,2)	(5,1)		6	5/36
(1,6)	(2,5)	(3,4)	(4,3)	(5,2)	(6,1)	7	6/36
(2,6)	(3,5)	(4,4)	(5,3)	(6,2)		8	5/36
(3,6)	(4,5)	(5,4)	(6,3)			. 9	4/36
(4,6)	(5,5)	(6,4)				10	3/36
(5,6)	(6,5)					11 .	2/36
(6,6)						12	1/36

With more combinations (six) yielding a total of seven than any other, seven is the most likely total to be thrown, with probability 6/36, 1/6, or 16.67%. Working outwards from seven in either direction, the totals become progressively less likely until you get to two and twelve, the least likely outcomes, each with probability 1/36 or 2.78%. This effect is portrayed in the above table by the pyramid shaped list of outcomes.

THE MAIN BETS

The main bets in craps are the pass line, the come, the don't pass, the don't come and odds.

PASS LINE BET

When a game or new round of play begins, the roll is called the "come-out" roll. A bet on the pass line wins even money if the come-out roll is a 7 or 11, and loses if it is a 2, 3, or 12 (a roll of 2, 3, or 12 is called "craps"). If any other number is rolled, this number becomes the "point." Once a point is established, the pass wager is not resolved until the point is rolled again or a 7 is rolled. If the point is rolled a second

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time before a 7, the pass line wager wins even money. If a 7 occurs before the point is rolled a second time, the pass line wager loses.

The house advantage on the pass line bet is 1.414%. To see how this is calculated, note that there are two ways to win this bet – winning the come-out roll or making the point. Since there are six possible points (4, 5, 6, 8, 9, and 10), the number of ways to win the pass line bet can be further broken down into seven possibilities:

- (1) win the come-out roll,
- or (2) establish and make the point 4,
- or (3) establish and make the point 5,
- or (4) establish and make the point 6,
- or (5) establish and make the point 8,
- or (6) establish and make the point 9.
- or (7) establish and make the point 10.

DICE HAVE THEIR LAWS, WHICH COURTS OF JUSTICE CANNOT UNDO.

ST. AMBROSE (D. 397)

Similarly, the pass line bet could be lost in seven distinct ways. To calculate the expectation, the probabilities for each of these outcomes are needed.

Since there are eight ways to roll a 7 or 11 (winning numbers for the pass line bet), the probability of winning the come out roll is 8/36, or .222222. Since there are 4 ways to roll a 2, 3, or 12, the probability of losing the come-out roll is 4/36, or .111111. The probability of establishing and making a point depends on the particular point number. For example, the probability of establishing 4 as the point is 3/36 (three ways to roll a 4), and once established, the probability of making (passing) that point is 3/9 (there are six ways to roll a 7 and three ways to roll a 4). Thus the probability of establishing then making the point 4 is (3/36)(3/9) = 1/36. The probability of establishing then not making the point four is (3/36)(6/9) = 2/36. Similar reasoning can be used to find the probabilities associated with the points 5, 6, 8, 9, and 10. The following table summarizes the probabilities:

Point Probabilities in Craps						
Point Establish		Establish then Pass	Establish then don't pass			
4	3/36	$3/36 \times 3/9 \approx .027778$	$3/36 \times 6/9 \approx .055556$			
5	4/36	4/36 x 4/10 ≈ .044444	4/36 x 6/10 ≈ .066667			
6	5/36	$5/36 \times 5/11 \approx .063131$	5/36 x 6/11 ≈ .075758			
8	5/36	$5/36 \times 5/11 \approx .063131$	5/36 x 6/11 ≈ .075758			
9	4/36	$4/36 \times 4/10 \approx .044444$	4/36 x 6/10 ≈ .066667			
10	3/36	$3/36 \times 3/9 \approx .027778$	$3/36 \times 6/9 \approx .055556$			

Since winning the pass line bet pays even money, the expectation on a 1-unit pass line wager is:

$$EV = (+1)(.222222) + (+1)(.027778) + (+1)(.044444) + (+1)(.063131)$$

$$+ (+1)(.063131) + (+1)(.044444) + (+1)(.027778) + (-1)(.111111)$$

$$+ (-1)(.055556) + (-1)(.066667) + (-1)(.075758) + (-1)(.075758)$$

$$+ (-1)(.066667) + (-1)(.055556)$$

$$= (+1)(.492929) + (-1)(.507071) = -0.01414.$$

This represents a 1.414% house advantage on the pass line bet. The calculation above also shows the overall probability of winning the pass line bet is .492929 and the overall probability of losing the pass line bet is .507071.

After a point is established, a bet on the pass line cannot be removed or reduced, although it may be increased. This latter move is unfavorable to the player since the advantage on the pass line bet is on the come-out roll and the odds are against the pass line bet once a point is established. Also unfavorable, a player who misses the come-out roll may make a bet on the pass line after a point is established (called a *put bet*).

FREE ODDS

The odds bet is called free odds because ... well, because it's free - the house advantage is zero. Odds can be taken after a point has been established on the pass line wager (odds can also be taken on come, don't pass, and don't come after a point is established - these bets are discussed on the next page). Like the pass line bet, if the point is rolled before a 7, the odds bet wins. If a 7 is rolled before the point, the odds bet loses. Unlike the pass line, which pays even money, a winning odds bet is paid according to the true odds. For example, suppose a \$5 pass line bet is made and a 4 is thrown on the come-out roll. Odds may now be taken on the point 4. The amount of odds that can be taken varies depending on the casino, but for this example let's assume single odds are taken. This is an additional \$5 bet that will win if a 4 is rolled before a 7 and lose if a 7 is rolled before a 4. Since there are six ways to roll a 7 and three ways to roll a 4, the true odds against winning this bet are 2 to 1. If the point passes (a 4 is rolled before a 7), the pass line will be paid \$5 (even money) and the odds bet will be paid \$10 (2 to 1). Because odds bets are paid according to the true odds, the expectation and house advantage are zero. For example, the expected value calculation for a \$10 odds bet on the point 4 is:

$$EV(\$10) = (+\$20)(1/3) + (-\$10)(2/3) = \$0.00.$$

Because the house advantage is zero on the odds bet, taking odds when a point is established will lower the overall house advantage on the total wager, pass line and odds bet combined, from the 1.414% edge on the pass line alone. With single odds, the house advantage on the combined total wager is 0.848%, with double odds 0.606%, with triple odds 0.471%, with 5X odds 0.326%, and with 10X odds 0.184%. Many casinos are now allowing a 3/4/5X odds structure, where the player may take 3X odds on the 4 and 10, 4X odds on the 5 and 9, and 5X odds on the 6 and 8. This

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3/4/5X odds structure leads to a 0.374% house advantage on the total wager (pass line or come plus odds).

The odds bet may be removed at any time. Come odds (see come bet below) are typically "off," or inactive, on a come-out roll, unless the bettor specifically asks that they be "on." Don't come odds are always on.

THE COME BET

The come bet gives the player the opportunity to make what is essentially a pass line bet after a point is established. A come bet turns the next roll of the dice into a come-out roll that wins on a 7 or 11, loses on a 2, 3, or 12, and otherwise establishes a point that will win if the point number is rolled again before a 7 and will lose if a 7 is rolled before the point. Since the come bet works like the pass line bet, the probabilities and resulting edge are the same as that for the pass line (1.414% with no odds, 0.848% with single odds, 0.606% with double odds, 0.471% with triple odds, 0.326% with 5X odds, 0.184% with 10X odds, and 0.374% for 3/4/5X odds).

THE DON'T PASS LINE BET

The don't pass wager works the opposite of the pass line bet, except that don't pass bettors are barred from winning on a come-out roll of 12. The don't pass line bet wins on a come-out roll of 2 or 3, loses on a 7 or 11, and ties on a 12. On point rolls, the don't pass wins if a 7 is rolled before the point and loses if the point is rolled before a 7. An analysis similar to the one for the pass line wager shows the house advantage on the don't pass bet to be 1.364%. 58

Like the pass line bet, odds can be taken once a point is established. Odds on the don't pass line bet will win if a 7 rolls before the point and are paid at true odds -1 to 2 for the 4 or 10, 2 to 3 for the 5 or 9, and 5 to 6 for the 6 or 8. Taking odds will reduce the house advantage on the combined wager, don't pass line plus odds, to 0.682% with single odds, 0.455% with double odds, 0.341% with triple odds, 0.227% with 5X odds, 0.124% with 10X odds, and 0.273% for 3/4/5X odds.⁵⁹

Don't pass bets can be removed or reduced after a point is established.

THE DON'T COME BET

The don't come bet is essentially a don't pass bet made after a point has been established. It works just like the don't pass and has the same house advantage (1.364% with no odds, 0.682% with single odds, 0.455% with double odds, 0.341% with triple odds, 0.227% with 5X odds, 0.124% with 10X odds, and 0.273% with 3/4/5X odds).

⁵⁸ This is house advantage per roll. Since rolling 12 results in a push, the house advantage may also be expressed as 1.403% per resolved bet. There are other games, notably baccarat, in which the house advantage for certain wagers may be expressed either including or excluding ties.

⁵⁹ These are house advantages per roll. The corresponding house advantages per resolved bet are 0.691%, 0.459%, 0.343%, 0.228%, 0.124%, and 0.274% for 1X, 2X, 3X, 5X, 10X, and 3/4/5X odds.

OTHER CRAPS BETS

The pass line, don't pass line, come, and don't come wagers, along with the odds bet, represent the main wagers in craps. There are a plethora of other wagers that can be made, all of which carry a higher house advantage, some significantly so. A brief summary of these wagers and their house advantages is given below.

PLACE AND DON'T PLACE BETS

A place bet on one of the point numbers, 4, 5, 6, 8, 9, or 10, is a bet that the number will roll before a 7. Payoffs are less than true odds, typically 9 to 5 for the numbers 4 and 10, 7 to 5 for the numbers 5 and 9, and 7 to 6 for the numbers 6 and 8. The house advantage is 6.67% for the 4 and 10, 4.00% for the 5 and 9, and 1.52% for the 6 and 8. A don't place bet is the opposite of a place bet – it's a bet that a 7 will roll before the number. Like place bets, these wagers are paid off at less than true odds. The don't place on a 4 or 10 pays 5 to 11 (true odds are 1 to 2), a don't place on a 5 or 9 pays 5 to 8 (true odds 2 to 3), and a don't place on a 6 or 8 pays 4 to 5 (true odds 5 to 6). House advantages are 3.03% for the 4 and 10, 2.50% for the 5 and 9, and 1.82% for the 6 and 8. Expectation calculations and resulting advantages are shown below.

Place 4, Place 10

$$EV(\$5) = (+9)(3/9) + (-5)(6/9) = -0.3333,$$

 $HA = 0.3333/5 = 0.0667, \text{ or } 6.67\%.$

Place 5, Place 9

$$EV(\$5) = (+7)(4/10) + (-5)(6/10) = -0.2000,$$

 $HA = 0.2000/5 = 0.0400, \text{ or } 4.00\%$

Place 6, Place 8

$$EV(\$6) = (+7)(5/11) + (-6)(6/11) = -0.0909,$$

 $HA = 0.0909/6 = 0.0152, \text{ or } 1.52\%. \sim$

Don't Place 4, Don't Place 10

$$EV(\$11) = (+5)(6/9) + (-11)(3/9) = -0.3333,$$

 $HA = 0.3333/11 = 0.0303, \text{ or } 3.03\%.$

Don't Place 5, Don't Place 9

$$EV(\$8) = (+5)(6/10) + (-8)(4/10) = -0.2000,$$

 $HA = 0.2000/8 = 0.0250, \text{ or } 2.50\%.$

⁶⁰ Don't place bets are not very common these days and not available in many casinos.

Don't Place 6, Don't Place 8

EV(\$5) = (+4)(6/11) + (-5)(5/11) = -0.0909,HA = 0.0909/5 = 0.0182, or 1.82%.

BUY AND LAY BETS

Buy and lay bets are similar to the place and don't place bets except that the payoffs are at true odds and a 5% commission is charged on the amount of a buy bet and on the possible winning amount of a lay bet. All buy bets have a 4.76% house advantage. Lay bets carry house advantages of 2.44% for the 4 and 10, 3.23% for the 5 and 9, and 4.00% for the 6 and 8. Expectation and resulting house advantage calculations are shown below.

Buy 4, Buy 10

EV(\$21) = (+\$39)(3/9) + (-\$21)(6/9) = -\$1.00,HA = 1.00/21 = 0.0476, or 4.76%.

Buy 5, Buy 9

EV(\$21) = (+\$29)(4/10) + (-\$21)(6/10) = -\$1.00,HA = 1.00/21 = 0.0476, or 4.76%.

Buy 6, Buy 8

EV(\$21) = (+\$23)(5/11) + (-\$21)(6/11) = -\$1.00,HA = 1.00/21 = 0.0476, or 4.76%.

Lay 4, Lay 10

EV(\$41) = (+\$19)(6/9) + (-\$41)(3/9) = -\$1.00,HA = 1.00/41 = 0.0244, or 2.44%.

Lay 5, Lay 9

EV(\$31) = (+\$19)(6/10) + (-\$31)(4/10) = -\$1.00,HA = 1.00/31 = 0.0323, or 3.23%.

Lay 6, Lay 8

EV(\$25) = (+\$19)(6/11) + (-\$25)(5/11) = -\$1.00,HA = 1.00/25 = 0.0400, or 4.00%. THE FIELD BET

The field bet is a bet that the next roll will be a 2, 3, 4, 9, 10, 11, or 12. The payoff is typically 2 to 1 for 2 and 12, and even money for 3, 4, 9, 10, and 11. Although this wager covers seven out of eleven numbers, the losing numbers (5, 6, 7, and 8) can be rolled more in more ways. The house advantage is 5.56%.

This can be seen with the usual expected value calculation for a \$1 wager:

$$EV = (+\$2)(2/36) + (+\$1)(14/36) + (-\$1)(20/36) = -\$0.0556.$$

Some casinos offer a 3 to 1 payoff on either the 2 or 12 (but not both). A calculation similar to the one above will show that paying 3 to 1 for either the 2 or 12 lowers the house advantage to 2.78%. (Paying 3 to 1 on both the 2 and 12 would result in a house advantage of zero.)

BIG 6 AND BIG 8

The big 6 is a bet that the 6 will roll before a seven; the big 8 is that the 8 will roll before a 7. The payoff is even money and the house advantage is 9.09%, as can be seen from the following expectation calculation:

$$EV = (+1)(5/11) + (-1)(6/11) = -0.0909.$$

The big 6 and big 8, the place 6 and place 8, and the buy 6 and buy 8 are exactly the same wagers but with different payoffs, and hence different house advantages. The place 6 and place 8 pay 7 to 6 and have a house advantage of 1.52%; the buy 6 and buy 8 pay the true odds of 6 to 5 but charge a 5% commission, resulting in a house advantage of 4.76%; the big 6 and big 8 pay even money and have a house advantage of 9.09%.

HARDWAYS

There are four hardway wagers: hard 4, hard 6, hard 8, and hard 10. A hardway is a wager that the selected number will roll with doubles before the number is rolled any other way or a 7 is rolled. For example, a hard 6 wins if 3-3 comes up before an easy 6 (1-5, 5-1, 2-4, or 4-2) or a 7. Payoffs are 7 to 1 for hard 4 and hard 10, and 9 to 1 for hard 6 and hard 8. Expected value calculations are straightforward. For the hard 4 or hard 10, there are eight losing combinations (two easy ways to roll the selected number – 1-3, 3-1 or 4-6, 6-4 – and six ways to roll a 7) and only one winning combination (either 2-2 or 5-5).

Thus the expectation is:

$$EV = (+7)(1/9) + (-1)(8/9) = -0.1111.$$

For the hard 6 or hard 8, there are ten losing combinations (four easy ways to roll the selected number -1-5, 5-1, 2-4, 4-2 or 2-6, 6-2, 3-5, 5-3 — and six ways to roll a 7) and only one winning combination (either 3-3 or 4-4). The expectation is:

$$EV = (+9)(1/11) + (-1)(10/11) = -0.0909.$$

House advantages are 11.11% for the hard 4 and hard 10, and 9.09% for the hard 6 and hard 8.

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ANY CRAPS

Any craps is a one-roll bet that the next roll will be a 2, 3, or 12. If the next roll is not a 2, 3, or 12, the wager is lost. The payoff is 7 to 1⁶¹ and the expected value is:

$$EV = (+7)(4/36) + (-1)(32/36) = -0.1111,$$

 $HA = 11.11\%.$

ANY SEVEN

Another one-roll bet, any seven wins 4 to 1 if the next roll is a 7. The expected value is:

$$EV = (+4)(6/36) + (-1)(30/36) = -0.1667,$$

 $HA = 16.67\%.$

CRAPS AND ELEVEN (C&E)

Craps and eleven (C&E) is a one-roll bet that the next roll will be a 2, 3, 11, or 12. Using payoffs of 3 to 1 on the 2, 3, and 12, and 7 to 1 on the 11, the expected value is:

$$EV = (+3)(4/36) + (+7)(2/36) + (-1)(30/36) = -0.1111,$$

 $HA = 11.11\%.$

TWO OR TWELVE

These are two separate one-roll bets – a wager can be made that the next roll will be a two or a wager can be made that the next roll will be a twelve. The payoff is typically 30 to 1 for each.⁶²

The expectation using this payoff is given by:

$$EV = (+30)(1/36) + (-1)(35/36) = -0.1389,$$

 $HA = 13.89\%$ (for either wager).

THREE OR ELEVEN

Like the wagers on two or twelve, these are two separate one-roll bets. Each typically pays 15 to 1.63

$$EV = (+15)(2/36) + (-1)(34/36) = -0.1111,$$

 $HA = 11.11\%$ (for either wager).

⁶¹ Often the payoffs on the proposition bets in craps are posted in terms of "for" rather than "to." The payoff for the any craps wager, for example, is 8 for 1.

⁶² Some casinos only pay 29 to 1, yielding a house advantage 16.67%.

⁶³ Some casinos only pay 14 to 1, yielding a house advantage 16.67%.

HORN BET

The horn is a one-roll bet that the next roll will be 2, 3, 11 or 12. It is really just four separate bets on these four numbers, so the bet is made in multiples of four. The horn returns the usual payoffs for the individual winning number: typically 30 to 1 for the 2 and 12, and 15 to 1 for the 3 and 11. With these payoffs and four units wagered on the horn, the bet will net 27 units for a hit on 2 or 12, and 12 units for a hit on 3 or 11. The expected value is:

$$EV = (+27)(2/36) + (+12)(4/36) + (-4)(30/36) = -0.50.$$

Since the bet was 4 units, the house advantage is 0.50/4 = 0.125, or 12.5%.

HORN HIGH BET

The high horn is a horn bet made in multiples of five with the extra unit wagered on the "high" number designated by the bettor. For example, a \$5 high twelve would put \$1 each on the 2, 3, and 11, and \$2 on the 12. With 30 to 1 payoffs for the 2 and 12, and 15 to 1 for the 3 and 11, the house advantage depends on which number is chosen to be "high." If 2 or 12 are designated high, the expected value is given by

$$EV = (+57)(1/36) + (+26)(1/36) + (+11)(4/36) + (-5)(30/36) = -0.6389.$$

If 3 or 11 are designated high, the expectation is:

$$EV = (+26)(2/36) + (+27)(2/36) + (+11)(2/36) + (-5)(30/36) = -0.6111.$$

These expectations yield house advantages of 12.78% (0.6389/5 = 0.1278) for the high two and high twelve, and 12.22% (0.6111/5 = 0.1222) for the high three and high eleven.⁶⁵

HOP BET

The hop is a one-roll bet that the next roll will be a certain combination. There are two types, an "easy" or two-way hop, such as 6-1, and a "hard" or one-way hop, such as 2-2.66 Typical payoffs are 15 to 1 for an easy hop and 30 to 1 for a hard hop. The expected value calculation for the easy hop is:

$$EV = (+15)(2/36) + (-1)(34/36) = -0.1111.$$

For the hard hop:

$$EV = (+30)(1/36) + (-1)(35/36) = -0.1389.$$

House advantages are 11.11% for the easy hop and 13.89% for the hard hop.⁶⁷

⁶⁴ If payoffs are 29 to 1 on the 2 and 12, and 14 to 1 on the 3 and 11, the house advantage is 16.67%.

⁶⁵ If payoffs are 29 to 1 on the 2 and 12, and 14 to 1 on the 3 and 11, the house advantages are 16.67% for all high horn bets.

⁶⁶ A hard hop differs from a hardway bet in that the hard hop is a one-roll bet whereas the hardway is decided only if the roll is the designated total, easy or hard, or a seven.

⁶⁷ Some casinos offer only 14 to 1 for an easy hop and 29 to 1 for a hard hop, raising the house advantages to 16.67% for both wagers.